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This paper describes an analysis of various sets of design effects constructed from the Census Bureau's March 1973 Current Population Survey (CPS). The paper is divided into five parts. In the first part we present the basic definitions, a discussion of our earlier results, and some limitations on the calculations to be performed. The second part is an investigation of some conjectures (of Kish and Frankel [1]), as they pertain to the CPS. In order to produce summary descriptors of collections of design effects, we consider, in part three, various schemes of averaging design effects. Since these schemes all appear to be unsatisfactory, in part four we propose an alternative type of summary descriptor based on the concept of empirical Stein estimation (see, for example, [2]). Finally, part five consists of a few brief concluding remarks.

## 1. INTRODUCTION AND BACKGROUND

**1.1 Design effects.**--Standard statistical methods have been developed under the assumption of simple random sampling (SRS). Although the independence of sample elements is often assumed, it is seldom realized in large complex surveys. As a result, practitioners [3, 4, 5] suggest alternative methods, such as jackknifing or the use of balanced repeated replication, for calculating sampling errors in complex surveys. Design effects [6] are essentially just measures for comparing such estimates of the "actual variance" to those computed under the SRS hypothesis.

In particular, for a given statistic  $X$ , we define the design effect of  $X$ ,  $\delta(X)$ , by

$$(1.1) \quad \delta(X) = \frac{\text{VAR}(X)}{\sigma^2(v)}$$

where  $\text{VAR}(X)$  is the (expected) variance of  $X$  for the actual complex survey, and  $\sigma^2(X)$  is the expected variance of  $X$  which would have been obtained by selecting, with replacement, a simple random sample of exactly the same size from the entire population surveyed. For example, if  $P$  is the actual proportion of items in a population with a given characteristic and  $n$  is the sample size, then the SRS variance of the usual estimator,  $\hat{P}$ , of  $P$  is

$$(1.2) \quad \sigma^2(\hat{P}) = P(1-P)/n.$$

The design effect,  $\delta$ , is a measure of the impact on the actual variance of the complexity of the sample design relative to that of simple random sampling; in other words,  $\delta$  summarizes the composite effect on the variance of such things as the number and nature of the selection at each stage of the sampling process, the extent of pre- and post-stratification, and the ultimate cluster size. We will use  $\delta$  to refer to population values and  $\hat{\delta}$  to sample values.

**1.2 Summary of previous results.**--In a paper delivered at the 1976 Annual Meeting of the American Statistical Association [7], Fritz Scheuren and I presented an empirical study that considered:

- (i) various methods of calculating individual design effects for proportions, and
- (ii) various methods of averaging these individual design effects.

The principal conclusions of that work were:

- (i) Each of the (asymptotically-equivalent) design effect estimators considered produced essentially the same value. (This suggests that, for our data, each estimator considered was equally good.)
- (ii) Different methods of averaging these design effects produced substantially diverse summary statistics.

The results on averaging methods in [7] warranted further examination and led directly to the present effort.

**1.3 Statistics considered.**--In last year's paper we considered design effects for CPS STATS units by race of the unit head.<sup>1/</sup> Within each racial group, design effects were calculated separately for five different classifiers: type of unit, total unit size, total earnings of unit, total social security benefits of unit, and total income of unit. The asymptotically unbiased estimators whose design effects we examined were

- (i)  $\hat{P}(W)$ , the proportion of whites in a given category, and
- (ii)  $\hat{P}(B)$ , the proportion of nonwhites in a given category. (Hereafter, we will refer to nonwhites as "blacks.")

In the present paper, we re-examine these design effects, as well as those of

- (iii)  $\hat{D} = \hat{P}(W) - \hat{P}(B)$ , the difference in the proportion of whites and "blacks" in a given category,
- (iv) Yule's  $Q$ , and
- (v) the cross-product ratio, denoted by  $C$ .

The last two statistics measure the association between the variables race (white or black) and inclusion (or exclusion) in a given category. In particular [9, p. 539], if we have the table of observed frequency counts

	White	Black
In category	a	b
Not in category	c	d

then Yule's Q and the cross-product ratio C may be estimated as

$$(1.3) \quad \tilde{Q} = \frac{ad-bc}{ad+bc}$$

and

$$(1.4) \quad \tilde{C} = \frac{bc}{ad} = \frac{1-\tilde{Q}}{1+\tilde{Q}}$$

This definition of the cross-product ratio is the reciprocal of the usual one. We use the symbol  $\delta(W)$  to denote the set of design effects for the proportion of whites. Similarly, we use  $\delta(B)$ ,  $\delta(D)$ ,  $\delta(Q)$ , and  $\delta(C)$  to denote, respectively, the set of design effects for the proportion of blacks, the difference in proportions, Yule's Q, and the cross-product ratio.

**1.4 Replicate estimators of design effects.--** There are, of course, many ways to construct estimators of design effects. In parts 2 and 3, we confine our attention to jackknife estimators which pertain when the sample may be separated into a number, say  $r$ , of independent, identically designed subsamples or replicates. The "replicates" employed in our study are the eight rotation panels of the March 1973 CPS.<sup>2/</sup> For a particular set of design effects, say  $\delta(W)$ , we will basically employ estimators of the form

$$(1.5) \quad \hat{\delta}(W) = \frac{\hat{VAR}(W)}{\hat{\sigma}^2(W)}$$

where

$\hat{VAR}(W)$  is the jackknife estimator of the actual variance of  $\hat{P}(W)$  and  $\hat{\sigma}^2(W)$  is an asymptotically unbiased estimator of the SRS variance of  $\hat{P}(W)$ .

Formulas for computing the SRS variance estimates considered here appear in [9] under the assumption that the sampling of blacks and whites is carried out independently. In particular, our estimate of  $\sigma^2(W)$  is

$$(1.6) \quad \hat{\sigma}^2(W) = \hat{P}(W) [1 - \hat{P}(W)] / n(W),$$

where  $n(W)$  denotes the total number of whites surveyed.

Estimates of all of the actual variances and some of the design effects are obtained by using the jackknifing technique. As in last year's paper, jackknifing is also used to calculate the standard errors of all the design effects considered.

**1.5 Some limitations.--** Because the same sample of PSU's is common to all rotation panels, it is not possible to use the panels to estimate the between-PSU component of the CPS variance. Con-

sequently, the "design effects" considered here relate only to the within-PSU component of the estimators. It might be mentioned, parenthetically, that for each statistic discussed in this paper, the within-PSU component probably accounts for at least 90 percent of the total variation.

The Census Bureau constructs all eight rotation panels in the same way. As already stated, we are using these 8 panels as the  $r=8$  replicates. Consequently, there is considerable variation (from 1 to 8) between panels (i.e., "replicates") in the number of times each of the interviewees is surveyed prior to and including the March 1973 interview.

Differences in the method of conducting the interviews also exist from panel-to-panel. Initially, the questions are asked in person; but, in the later panels, most of the surveying is done by telephone. The net effect of these and other factors [10] is to alter the response patterns from panel-to-panel so that the panels cannot be assumed to be *a priori* identically distributed. The influence of these panel differences on the statistics under consideration here is not known.<sup>3/</sup> When we began this work, we implicitly assumed that such panel effects, if any, would be small enough to ignore. This was in part, a reflection of our, perhaps misplaced, confidence in the nature of the raking ratio estimation procedures employed.<sup>4/</sup> Project plans call for a repetition of the present calculations using a random group estimator (described in [12]) that would not be subject to "panel biases."

## 2. AN EMPIRICAL COMPARATIVE INVESTIGATION OF SOME DESIGN EFFECT ESTIMATORS

**2.1 Kish-Frankel conjectures.--** This part of the paper is inspired by some conjectures of Kish and Frankel [1; p. 13]. Having defined  $\bar{Y}$  as the mean of the vector of statistics  $\underline{Y}$  and  $A$  as a complex function of  $\underline{Y}$ , we may list the Kish-Frankel conjectures as

- (i)  $\delta(A) > 1$ . In general, the population values of the design effects of complex statistics tend to be greater than 1.
- (ii)  $\delta(A) \leq \delta(\bar{Y})$ . The design effect of the mean  $\bar{Y}$  of a statistic  $\underline{Y}$  tends to be greater than those of complex functions of  $\underline{Y}$ .
- (iii)  $\delta(A)$  is related to  $\delta(\bar{Y})$ . For variates with high  $\delta(\bar{Y})$ , values of  $\delta(A)$  tend also to be high.
- (iv)  $\delta(A)$  tends to resemble the design effect for differences of means.
- (v)  $\delta(A)$  tends to have observable regularities for different statistics.

A simple model of the above would be

$$(2.1) \quad \delta(A_g) = 1 + f_g [\delta(\bar{Y}) - 1], \text{ with } \delta(\bar{Y}) > 1$$

$$\text{and } 0 < f_g < 1 \text{ and } f_g$$

specific to the variables and statistic denoted by  $g$ .

The calculations in this part of the paper are performed for both the original five "basic" sets of design effects and for "high-proportion" sets which are created by deleting from the basic sets those categories in which either the proportion of whites is less than 2% or the proportion of blacks is less than 5%.

**2.2 Conjecture (i).**--The first conjecture we examine is that the values of the design effects tend to be larger than 1. For the basic sets, each composed of 63 individual design effects, we find that 74.60% of the elements of  $\delta(W)$  are greater than 1. However, none of the other sets of design effects, including  $\delta(B)$ , shares this property; only about 50% of these values tend to be larger than 1.

For the high-proportion sets, each composed of 32 individual design effects, 75% of the elements of  $\delta(W)$  are larger than 1. Moreover, for the other four high-proportion sets, the percentage of values greater than 1 increases, although remaining somewhat below that of  $\delta(W)$ .

**2.3 Conjecture (ii).**--We next compare the values of the individual design effects of each set to the corresponding values of each of the other sets of design effects. For the basic sets, we find that the elements of  $\delta(W)$  tend to be larger (in about two-thirds of the cases) than the corresponding elements of the other sets of design effects. For example, 63.49% of the elements of  $\delta(W)$  exceed the corresponding values of  $\delta(B)$ . For the other four basic sets, no one set particularly dominates any other. For the high-proportion cases, the values of  $\delta(W)$ , again, tend to be the largest. The values of  $\delta(B)$  tend to be less than those of the three complex statistics; the values of  $\delta(D)$  and  $\delta(Q)$  are both generally less than those of  $\delta(C)$ .

In light of conjecture (ii), it is not surprising that the values of  $\delta(W)$  dominate the values of the other sets; however, it is, at least at first glance, surprising that the values of  $\delta(B)$  tend to be smaller than the values of the sets corresponding to the three complex statistics.

**2.4 Conjectures (iii), (iv), and (v).**-- We next examine the correlation coefficient of each pair of sets of design effects. We find, for the basic sets,  $\delta(W)$  is positively correlated with  $\delta(D)$  and negatively correlated with  $\delta(B)$ ,  $\delta(Q)$  and  $\delta(C)$ , the value of each of these four correlation coefficients being relatively close to zero; i.e., .0238, 0.0871, -.0502, and -.0495, respectively. For the high-proportion sets,  $\delta(W)$  is, again, negatively correlated with each of the other four sets, but here the magnitude of each

of these correlation coefficients is relatively large.

Excluding  $\delta(W)$ , the remaining four sets are very strongly positively correlated. This is not surprising. Since about eight times as many whites are surveyed as blacks, the blacks account for roughly 85% of the variance of the difference in proportions. Furthermore, since

$$(2.2) \quad 0 < \frac{2P(W) \cdot P(B)}{P(W) + P(B)} < 1,$$

we may write  $Q$  as

$$(2.3) \quad Q = \frac{P(W) - P(B)}{P(W) + P(B)} \left[ 1 + \frac{2P(W)P(B)}{P(W) + P(B)} + \left( \frac{2P(W)P(B)}{P(W) + P(B)} \right)^2 + \dots \right].$$

Also, since  $-1 \leq Q \leq 1$ ,

we may write  $C$  as

$$(2.4) \quad C = (1 - 2Q + 2Q^2 - 2Q^3 + \dots).$$

So  $C$  may be approximated by  $1 - 2Q$ , especially when the magnitude of  $Q$  is small. Thus,  $C$  is approximately a linear function of  $Q$ , and  $Q$  is approximately a linear function of the difference in proportions. It is, therefore, reasonable that the design effects for  $\delta$ ,  $Q$  and  $C$  tend to be nearly equal and are so high correlated.

Thus, considering the difference in proportions, Yule's  $Q$  and the cross-product ratio as complex functions of the proportion of blacks, we have an even stronger result than conjecture (iii); namely, that the design effects of (certain) complex statistics are highly-correlated with the design effects of the proportion of blacks.

### 3. ORIGINAL DESIGN EFFECT AVERAGING SCHEMES

In this part of the paper we reconsider the averaging schemes employed in our earlier paper. These schemes are applied to a number of sets of design effects not considered previously. Our goal here is to discover a good summary descriptor of sets of design effects.

In our earlier paper, we considered four types of "averages"--the median and three means (arithmetic, harmonic, and geometric). We also employed three distinct weighting schemes--uniform weighting, weighting by the reciprocal of the estimated simple random sampling (SRS) variances, and weighting by the reciprocal of the estimated SRS relvariances. Applying these  $4 \times 3 = 12$  averaging schemes to our five basic sets of design effects, we obtain the data of table 1.

The results of two additional averaging schemes are also shown in table 1. The first scheme, suggested by Kish, is the square of the average

of the square roots of the individual design effect estimates. Kish [6, p. 578] prefers this scheme. The second, referred to as the overall ratio average, is the average of all of the individual estimated actual variances divided by the corresponding average of the estimated SRS variances.

Last summer we were rather surprised that our 14 averaging techniques produced such diverse numerical results as those displayed in table 1. Consequently, we have since examined these calculations in much greater detail.

The most striking phenomenon concerned the two sets of non-uniform weighting schemes. In several instances, a relatively small number of the individual classes under consideration accounted for the predominant share of the weight. Consequently, in these cases, the values of the vast majority of the design effects of a particular set had almost no influence on the value of the summary statistic produced.

It is instructive at this point to consider a specific case: the relvariance weighting scheme applied to the estimators of the design effects of the proportion of blacks. In this particular instance, three of the 63 classes account for over 70% of the weight. These three classes are:

- (i) STATS units receiving no social security benefits (25.24%);
- (ii) STATS units having total earnings of less than \$10,000 (23.24%); and
- (iii) STATS units having total income of less than \$10,000 (22.11%).

In our earlier paper we also attempted to partition the sets of design effects into subsets which would be more homogeneous. In particular, the estimated design effects for the proportion of whites and blacks were partitioned into three or four groupings according to the estimated value of the corresponding proportion. This procedure narrowed the range of the averages substantially; however, the numerical differences among the various averaging schemes were still "uncomfortably" large.

#### 4. STEIN ESTIMATORS OF DESIGN EFFECTS

In light of the diverse results of the averaging schemes just presented, we decided to consider another method of constructing a summary descriptor of a set of design effects. The approach taken is discussed in Geisser [13, 14] and is based upon an empirical Stein-type estimator. Geisser's approach is of a heuristic, ad hoc nature. Its justification lies in whether or not it works in a given situation. We believe that such a scheme can be profitably applied to our data.

The Stein estimator, as originally formulated [2], requires a number of stringent assumptions,

some of which are clearly not valid in the present situation. On the other hand, Efron and Morris [15], among others, argue that the violation of these assumptions does not necessarily diminish the estimator's usefulness.

In the remainder of the paper we will discuss these issues as they pertain to our CPS data. The reader should keep in mind that we are only attempting to do some "dallying" with a few sets of design effects and are not attempting to resolve any of the outstanding theoretical issues concerning the general applicability of empirical Stein estimation.

4.1 Original Stein estimator.--We present here a brief description of Stein estimation. We, first, let  $j=1, \dots, J$  and  $k=1, \dots, K$  where  $K \geq 3$ . For a given collection of parameters  $\{\theta_j\}$ , we

assume that the random variables  $\{x_{kj}\}$  are independent and normally distributed with means  $\{\theta_j\}$  and common variance  $\sigma^2$ . In this setting, we define the Stein estimator of  $\theta_j$  to be

$$(4.1) \quad \hat{x}_j = \mu x_{..} + (1-\mu)x_{.j}$$

$$\text{where } x_{.j} = \frac{1}{K} \sum_{k=1}^K x_{kj} \quad \text{and}$$

$$(4.2) \quad x_{..} = \frac{1}{J} \sum_{j=1}^J x_{.j}$$

The unknown parameter " $\mu$ " is such that

$$(4.3) \quad 0 \leq \mu \leq 1.$$

James and Stein [2] have shown that for an appropriate choice of  $\mu$ , the use of the estimator  $\hat{x}_j$  produces, on the average, a smaller mean square error than the maximum likelihood estimator.

Following Geisser [13], we let  $\min(\mu_1, 1)$  be an estimator of  $\mu$  where

$$(4.4) \quad \mu_1 = \frac{(JK-1) m_1}{(J-1)m_1 + (K-1)Jm_2} \\ = \left[ \frac{(J-1)}{(JK-1)} + \frac{(K-1)J}{(JK-1)} \frac{m_2}{m_1} \right]^{-1}$$

with

$$(4.5) \quad m_1 = \frac{1}{J(K-1)} \sum_{k=1}^K \sum_{j=1}^J (x_{kj} - x_{.j})^2 \quad \text{and}$$

$$(4.6) \quad m_2 = \frac{K}{J-1} \sum_{j=1}^J (x_{.j} - x_{..})^2.$$

4.2 Stein estimation of design effects.--It now remains to relate the above formulation to the problem at hand. To limit the amount of computation involved, we restrict our attention to

$\tilde{\delta}(w) = \{\tilde{\delta}_j(w)\}$ , the set of computed design

effects for the proportion of whites. In addition, we only consider the harmonic and Kish (unweighted) averaging schemes, as these produced quite diverse results when applied to the individual  $\hat{\delta}_j(w)$ . (See table 1.)

Our first approach is to replace

- (i) the  $\{\Theta_j\}$  by  $\{\delta_j(w)\}$ , the actual (expected) values of the white design effects,
- (ii) the  $\{x_{.j}\}$  and  $\{\hat{x}_j\}$  by  $\{\tilde{\delta}_j(w)\}$  and  $\{\hat{\delta}_j(w)\}$  respectively, and
- (iii)  $x_{.j}$  by  $\tilde{\delta}_{.}(w)$ , an appropriate (i.e., harmonic or Kish) average of the set of individual design effects.

Noting that  $m_1$  equals  $K$  times the average of the usual estimators of the variances of the  $\{x_{.j}\}$ , we can write the " $m_1$ " and " $m_2$ " values corresponding to the  $\tilde{\delta}_j(w)$  as

$$(4.7) \quad \tilde{m}_1 = \frac{K}{J} \sum_{j=1}^J \sigma^2(\hat{\delta}_j(w))$$

and

$$(4.8) \quad \tilde{m}_2 = \frac{K}{J-1} \sum_{j=1}^J (\tilde{\delta}_j(w) - \tilde{\delta}_{.}(w))^2.$$

Hence, we have

$$(4.9) \quad \frac{\tilde{m}_2}{\tilde{m}_1} = (1-1/J)^{-1} \left[ \frac{\text{Mean squared deviation of the } \tilde{\delta}_j(w) \text{ from the overall average}}{\text{Mean of the estimated variances of the } \{\hat{\delta}_j(w)\}} \right].$$

This ratio can be evaluated by using the desired overall average  $\tilde{\delta}_{.}(w)$ , together with the  $\{\tilde{\delta}_j(w)\}$ , and the corresponding jackknife estimated variances.

It is now possible to estimate  $\mu_1$  by substituting  $\frac{\tilde{m}_2}{\tilde{m}_1}$  for  $\frac{m_2}{m_1}$  in equation (4.4) and choosing

suitable values for  $J$  and  $K$ . The choice of  $K=8$  and  $J=63$  is not optimal because it ignores the facts that the  $\{\tilde{\delta}_j(w)\}$ , while based on 8 independent replicates, are not sample means, and that the design effect for each of our 63 categories is not independent of those of the other categories.

Several ad hoc "solutions" to this selection problem were considered.<sup>5/</sup> The one which seems most reasonable to us is to note [13] that

$$(4.10) \quad \mu = \min(\mu_1, 1) \geq \min\left(\frac{\tilde{m}_1}{\tilde{m}_2}, 1\right)$$

and to simply choose  $\min\left(\frac{\tilde{m}_1}{\tilde{m}_2}, 1\right)$  as our

estimate of  $\mu$ . Computing a value for  $\mu$  in this manner, we find that it equals 1 for both the Kish and harmonic averaging schemes. This result was rather disappointing in that it leaves us exactly where we were at the end of section 3, with different averaging schemes producing diverse numerical results and no way to choose among them.

We suspect that the value of  $\mu = 1$  results from the heteroscedastic nature of the variances of the  $\hat{\delta}_j(w)$ . In order to "eliminate" this source of concern, we redefine  $\tilde{m}_1$  and  $\tilde{m}_2$  as

$$(4.11) \quad \tilde{m}_1 = \frac{K}{J} \sum_{j=1}^J \frac{\sigma^2(\hat{\delta}_j(w))}{\tilde{\delta}_j^2(w)}$$

and

$$(4.12) \quad \tilde{m}_2 = \frac{K}{J-1} \sum_{j=1}^J \frac{(\tilde{\delta}_j(w) - \tilde{\delta}_{.}(w))^2}{\tilde{\delta}_{.}^2(w)}.$$

Estimating  $\mu$  this time as  $\min\left[\frac{\tilde{m}_1}{\tilde{m}_2}, 1\right]$ , we

produce  $\mu$  values of 0.5040 and 0.8885 for the harmonic and Kish schemes, respectively. Using these values of  $\mu$ , we may re-estimate the white design effects by

$$(4.13) \quad \tilde{\delta}_j(w) = \mu \tilde{\delta}_{.}(w) + (1-\mu) \tilde{\delta}_j(w).$$

Our next task is to compare the two sets of design effects calculated from equation (4.13). Our approach involves calculating, for each averaging scheme, the nominal length of the symmetric 95% confidence interval of the proportion of whites in each of the 63 categories. This is done under the assumption <sup>6/</sup> that

$$E \hat{\hat{\delta}}_j(W) = \delta_j(W),$$

where  $\hat{\hat{\delta}}_j(W)$  is the estimator corresponding to  $\hat{\delta}_j(W)$ .

It can be shown that under regularity conditions the length of the  $j$ -th interval is proportional to

$$(4.14) \quad \sqrt{\hat{\delta}_j(W)} \cdot t(.95, DF(j)),$$

where  $t(.95, DF(j))$  is the length of a symmetric 95% confidence interval for a random variable having a Student's  $t$  distribution with

$DF(j)$  degrees of freedom.  $DF(j)$  is determined from

$$(4.15) \quad DF(j) = 2/\text{relvariance}(\hat{\delta}_j(W)).$$

Using this procedure, we find that, under Stein estimation, the length of the average confidence interval corresponding to the harmonic mean is 100.8% of that of the Kish mean. This compares to a value of 90.52% for the corresponding (unadjusted) averaging schemes of section 3. (This last quantity is simply the square root of the ratio of the harmonic average of the white design effects to that of the Kish average.)

Since the Stein estimation procedure has produced confidence intervals whose average lengths are more nearly equal under the harmonic and Kish schemes, the Stein technique appears to compare favorably, at least for our data, to the unadjusted overall averaging schemes of section 3. It should be pointed out, however, that we have gone only a very small part of the way towards applying Stein estimation to design effects. The chief difficulty, not addressed in this paper, is that the confidence intervals are, in general, biased; hence, in some situations, they could be very badly mis-estimated.

## 5. SOME CONCLUDING REMARKS

First and foremost, we must, again, emphasize that we have performed an empirical examination of the data of a single sample survey. In addition, we have only considered a very limited number of statistics. In general, the analysis described in part two confirms the conjectures of Kish and Frankel [1; p. 13].

As in our earlier paper, the averaging schemes discussed in part 3, unfortunately, produced widely diverse results. This may be because the sets of design effects considered were not sufficiently homogeneous for some or all of the averaging methods. On the other hand, the empirical Stein estimation scheme described in part four produced somewhat better results;

i.e., the lengths of the confidence intervals were on the average more nearly equal. These improved results were, however, obtained by the application of an ad hoc technique to a single set of data. Thus, our evidence in support of the Stein estimation scheme is not exactly overwhelming.

There is no doubt that much theoretical work is needed to "resolve" the issues raised here. As Kish and Frankel [1, p. 13] suggest, such theory will need to be buttressed by empirical results. We, therefore, encourage others to do some "dallying" with their own favorite sets of design effects, as we will continue to do ourselves.

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- 1/ A "STATS" unit is a group of individuals in a CPS household who would generally be considered to be interdependent under social insurance programs. The STATS unit concept is defined in [8].
- 2/ See subsection 1.5 below for the limitation imposed by this use of rotation panels as replicates.
- 3/ It should be pointed out, however, that to the extent that there are any panel differences, these would lead to an increase in the expected value of the estimated design effects.
- 4/ The estimator being used is described in [11] where it is referred to as the "intermediate undercount raking weight."
- 5/ One such "solution" involves letting

$$\mu = Km_1 / [(K-1)m_2 + m_1] \quad \text{with } K=7.$$

- 6/ We have some results for the more realistic and interesting case when  $E\hat{\delta}_j \neq \delta_j(W)$ , but these were not presented at the session and, in any case, are still incomplete.

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Table 1.--Selected methods of averaging CPS within-PSU design effects: Usual and jackknifed estimators of the design effects, standard error and coefficient of variation of averages

Item	Jackknife Estimator					Coefficient of Variation				
	Proportion of whites	Proportion of blacks	Difference in proportions	Yule's Q	Cross-Product Ratio	Proportion of whites	Proportion of blacks	Difference in proportions	Yule's Q	Cross-Product Ratio
Uniform unit weighting:										
Arithmetic.....	1.6200	1.2574	1.2384	1.2509	1.2160	.1545	.0714	.1124	.0930	.0977
Geometric.....	1.4243	.9964	1.0106	1.0183	.9945	.1513	.0653	.1244	.0908	.0932
Harmonic.....	1.2440	.7233	.3040	.8218	.7978	.1737	.1974	.1849	.1132	.1163
Median.....	1.4249	1.0292	1.0745	.9646	.9920	.1008	.1736	.1726	.1300	.1196
Weighting by reciprocal of estimated variances under simple random sample:										
Arithmetic.....	1.3819	1.2263	1.1231	1.2673	1.1046	.0631	.1208	.1515	.0950	.0332
Geometric.....	1.2443	.9156	.8866	1.0657	.9519	.1153	.0697	.1744	.1097	.1322
Harmonic.....	1.1276	.5615	.6765	.8997	.8071	.1891	.3291	.2565	.1254	.1433
Median.....	1.2664	.9474	.9326	1.0219	.9920	.1967	.2214	.2811	.1614	.3939
Weighting by reciprocal of estimated reliabilities under simple random sample:										
Arithmetic.....	2.2627	.9146	1.3046	1.2259	1.2728	.3721	.1276	.2119	.1469	.1079
Geometric.....	2.0254	.6402	1.1722	1.0827	1.3668	.3197	.1789	.2620	.2102	.1077
Harmonic.....	1.7791	.7952	.9984	.9506	.8962	.2869	.2116	.2835	.2203	.1791
Median.....	2.2711	.7550	1.2466	1.0219	.9920	.5310	.3477	.4404	.3688	.1571
Kish approach.....	1.5201	1.1260	1.1230	1.1303	1.1033	.1504	.0625	.1122	.0889	.0922
Overall ratio average.....	1.9517	1.3103	1.3160	1.1570	.8377	.3275	.1017	.1133	.1128	.1214

Note: The values shown for proportions in this table differ somewhat from the corresponding values shown in last year's paper because, even though the same data set was used, the way we defined the categories was altered slightly.